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STUDY OF AN INVENTORY MODEL FOR DECAYING ITEMS WITH STOCK DEPENDENT DEMAND AND EXPONENTIAL PRODUCTION RATE

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ABSTRACT

In the present paper a volume flexible manufacturing system is considered for a decaying item with an inventory-level-dependent demand rate. In reality, the demand rate remains stock-dependent for some time and then becomes a constant after the stock falls down to a certain level. Many factors like limited number of potential customers and their goodwill, price and quality of the goods, locality of shop, etc. can be accounted for the change in the demand pattern.

INTRODUCTION

Inventory is a part of every fact of business life. Without inventory any business can not be performed, whether it being service organization. Under increased competition, inventory based business are forced to better co-ordinate their procurement and marketing decisions to avoid carrying excessive stock when sales are low or shortages when demand are high. An effective means of such co-ordination is to conduct the inventory control and manufacturing decision jointly. The main task is to determine the optimal rate of production and inventory policy for a given time varying demand.

In the Classical Economic Production Lot Size(EPLS) model, the production rate of a machine is regarded to be pre-determinded and inflexible1.Alder and Nanda (1974), Sule (1981), Axsater and Elmaghraby (1981), Muth and Spearmann (1983) extended the EPLS model to situations where learning effects would induce an increase in the production rate. Proteus (1986), Rosenblat and Lee (1986) and Cheng (1991) considered the EPLS model in an imperfect production process in which the demand would exceed the supply. Schweitzer and Seidmann (1991) adopted, for the first time, the concept of flexibility in the machine production rate and discussed optimization of processing rates for a FMS (flexible manufacturing system). Obviously, the machine production rate is a decision variable in the case of a FMS and then the unit production cost becomes a function of the production rate. Khouja and Mehrez (1994) and Khouja (1995) extended the EPLS model to an imperfect production process with a flexible production rate. Silver (1990), Moon, Gallego and Simchi-Levi (1991) discussed the effects of slowing down production in the context of a manufacturing equipment of a family of items, assuming a common cycle for all the items. Gallego (1993) extended this model by removing the stipulation of a common cycle for all the items. But the above studies did not consider the demand rate to be variable. It is a common belief that large piles of goods displayed in a supermarket lead the customers to buy more. Silver and Peterson (1985) and Silver (1979) have also noted that sales at the retail level tend to be proportional to the inventory displayed. Baker and Urban (1988) and Urban (1992) considered an inventory system in which the demand rate of the product is a function of the on-hand inventory. Goh (1994) discussed the model of Baker and Urban18 relaxing the assumption of a constant holding cost. Mandal and Phaujder (1989) then extended this model to the case of deteriorating items with a constant production rate. Datta and Pal (1990) presented an inventory model in which the demand rate of an item is dependent on the on-hand inventory level until a given inventory level is achieved, after which the demand rate becomes constant. Giri , Pal , Goswami and Chaudhuri (1995) extended the model of Urban (1992) to the case of items deteriorating overtime. Ray and Chaudhuri (1997) discussed an EOQ (economic order quantity) model with stock-dependent demand, shortage, inflation and time discounting of different costs and prices associated with the system. Ray, Goswami and Chaudhuri (26 studied the inventory problem with a stock-dependent demand rate and two levels of storage, rented warehouse (RW) and own warehouse (OW). Giri and Chaudhuri (1998) extended the model of Goh (1994) to cover an inventory of a deteriorating item and discussed both the case of nonlinear time-dependent and stock-dependent holding costs.

ASSUMPTIONS AND LIMITATIONS

The mathematical model in this paper is developed on the basis of the following notations.

1. The time-horizon is infinite.
2. The inventory system involves only one item and is a self-production system.
3. No shortages are permitted.
4. Lead time is zero.
5. The production rate is considered to be a decision variable.
6. The production cost per unit item is a function of the production rate.

NOTATIONS

The mathematical model in this paper is developed on the basis of the following notations.

P :$P = a.e^{bt}$, the exponentially increasing production rate with respect to time 't'. Here a & b are positive constants and $a \geq b$.
I(t) :On-hand inventory at time 't' ≥ 0 .
R(I) :Demand-rate function varying with I(t)
η(P) : The production cost per unit item.
S_p : Salvage cost per unit item.
T : The duration of the production cycle.
∇ : Gradient operator.
S : The stock-level, beyond which the demand rate becomes constant.
θ : Constant deterioration rate of the On-hand inventory,$0 < \theta < 1$.
C_h : Holding cost per unit per unit time.
C_s : Setup cost per production run.

FORMULATION OF THE MODEL

We consider a self-manufacturing system in which the items are manufactured in a machine and the market demand is filled by these manufactured items. The demand rate is dependent on the on-hand inventory down to a level S_0, beyond which it is assumed to be a constant, i.e.,

$R(I) = D + \gamma I(t), I > S_0$

$= D + S_0, 0 \leq I \leq S_0,$

Here D and γ are non-negative constants and D < P.

The production cost per unit is

$$\eta(P) = r + \frac{g}{p} + \alpha P$$

Where r, g, and α are all positive constants. This cost is based on the following factors:

1. The material cost r per unit item is fixed.
2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the production cost per unit (g/P) decreases as the production rate (P) increases.
3. The third term (α P), associated with tool/die costs, and is proportional to the production rate.

MATHEMATICAL FORMULATION AND ANALYSIS OF THE MODEL

The production cycle begins with zero stock. Production starts at t = 0, and the stock reaches a level S_0 at time $t = t_1$ after meeting demands. The demand rate in the interval $(0, t_1)$ is $(D+\gamma S0)$. In the interval

(t_1, t_2),production continues uninterruptedly and the demand rate depends on the instantaneous stock level. Production is stopped at time $t = t_2$. The demand rate continues to depend on the instantaneous inventory level until $t = t_3$ when the stock falls down to the level S_0 again. The inventory falls to the zero level at the end t = T of the production cycle. This cycle of production is repeated over and over again. Therefore, the governing equations of this model are given by

$$\frac{dI(t)}{dt} + \theta\, I(t) = a.e^{bt} - (D+\gamma S_0),\ 0 \le t \le t_1,\ I \le S_0,\ b = 0.$$

With $I(0) = 0$ and $I(t_1) = S_0$ (1)

$$\frac{dI(t)}{dt} + (\theta+\gamma)\, I(t) = a.e^{bt} - D,\ t_1 \le t \le t_2,\ I > S_0,\ b = 0. \quad (2)$$

$$\frac{dI(t)}{dt} + (\theta+\gamma)\, I(t) = -D,\ t_2 \le t \le t_3,\ I > S_0,\ \text{with } I(t_3) = 0 \quad (3)$$

$$\frac{dI(t)}{dt} + \theta\, I(t) = -(D + \gamma S_0),\ t_3 \le t \le T,\ I \le S_0,\ \text{with } I(t) = 0. \quad (4)$$

From (1)

$$\frac{dI(t)}{dt} + \theta\, I(t) = a.e^{bt} - (D+\gamma S_0)$$

The solution of the above equation by applying boundary conditions I(0)=0, is given by

$$I(t).\ e^{\theta t} = \frac{a.e^{(b+\theta)t}}{(b+\theta)} - \frac{(D+\gamma S_0)\, e^{\theta.t}}{\theta} - \frac{a}{(b+\theta)} + \frac{(D+\gamma S_0)}{\theta}$$

Given b=0, putting b=0

$$I(t).\ e^{\theta t} = \frac{a.e^{\theta.t}}{\theta} - \frac{(D+\gamma S_0)\, e^{\theta.t}}{\theta} - \frac{a}{\theta} + \frac{(D+\gamma S_0)}{\theta}$$

$$I(t) = \left(\frac{a - D - \gamma S_0}{\theta}\right)(1- e^{\theta t}) \quad (5)$$

Now the applying the boundary conditions $I(t_1)=S_0$

Putting $t=t_1$, we get

$$I(t_1) = \left(\frac{a - D - \gamma S_0}{\theta}\right)\left(1 - e^{-\theta.t_1}\right)$$

$$S_0 = \left(\frac{a - D - \gamma S_0}{\theta}\right)(1 - e^{-\theta.t_1})$$

$$.t_1 = -\frac{1}{\theta}\left(1 - \frac{S_0\theta}{a - D - \gamma S_0}\right) \quad (6)$$

from (2)

$$\frac{dI(t)}{dt} + (\theta+\gamma)\, I(t) = a.e^{bt} - D$$

The solution by applying boundary conditions $I(t_1) = S_0$

$$I(t)\, e^{(\theta+\gamma)t} = \frac{a.e^{(b+\theta+\gamma)t}}{(b+\theta+\gamma)} - \frac{D.e^{(\theta+\gamma)t}}{(\theta+\gamma)} + S_o\, e^{(\theta+\gamma)t_1} - \frac{a.e^{(b+\theta+\gamma)t_1}}{(b+\theta+\gamma)} + \frac{D.e^{(\theta+\gamma)t_1}}{(\theta+\gamma)}$$

Putting b=0 we get

$$I(t) = \left(S_0 - \frac{a-D}{(\theta+\gamma)}\right)e^{(\theta+\gamma)(t_1-t)} + \left(\frac{a-D}{\theta+\gamma}\right) \qquad (7)$$

Therefore t= t_2 putting we get

$$I(t_2) = \left(S_0 - \frac{a-D}{\theta+\gamma}\right)e^{(\theta+\gamma)(t_1-t_2)} + \left(\frac{a-D}{\theta+\gamma}\right) \qquad (8)$$

From (3) $\frac{dI(t)}{dt} + (\theta+\gamma)I(t) = -D$

solution of the above equation by applying boundary conditions is

$$I(t) = I(t_2) + \frac{D}{(\theta+\gamma)})e^{(\theta+\gamma)(t_2-t)} - \frac{D}{(\theta+\gamma)} \qquad (9)$$

Now $I(t_3) = S_0$, Putting $t=t_3$ we get

$$t_3 = t_2 - \frac{1}{(\theta+\gamma)} \log\left[\frac{S_0 + \frac{D}{(\theta+\gamma)}}{I(t_2) + \frac{D}{(\theta+\gamma)}}\right] \qquad (10)$$

from (4)

$$\frac{dI(t)}{dt} + \theta.I(t) = -(D+\gamma.S_0)$$

Solution of the above equation with boundary conditions $I(t_3) = S_0$ is given by

$$I(t) = \left(S_0 + \frac{(D+\gamma.S_0)}{\theta}\right)e^{\theta.(t_3-t)} - \frac{(D+\gamma.S_0)}{\theta} \qquad (11)$$

Now I(t) = 0

$$T = t_3 - \frac{1}{\theta}\log\left(\frac{D+\gamma.S_0}{D+S_0(\theta+\gamma)}\right) \qquad (12)$$

let Inv1, Inv2, Inv3, Inv4 be the total inventories in the intervals $0 \le t \le t_1$, $t_1 \le t \le t_2$, $t_2 \le t \le t_3$, $t_3 \le t \le T$, Respectively. Then

$$Inv_1 = \int_0^{t_1} I(t)dt = \left(\frac{a-D-\gamma.S_0}{\theta^2}\right)\left[\theta.t_1 + e^{-\theta.t_1} - 1\right]$$

$$Inv_2 = \int_{t_1}^{t_2} I(t)dt$$

$$= \frac{1}{(\theta+\gamma)}\left(S_0 - \frac{a-D}{(\theta+\gamma)}\right)\left\{1 - e^{(\theta+\gamma)(t_2-t_1)}\right\} + \frac{a-D}{(\theta+\gamma)}\left[t_2 - t_1\right]$$

$$\text{Inv}_3 = \int_{t_2}^{t_3} I(t)dt = \frac{1}{(\theta+\gamma)}\left(I(t_2)+\frac{D}{(\theta+\gamma)}\right)\left[1-e^{(\theta+\gamma)(t_2-t_3)}\right]-\frac{D}{(\theta+\gamma)}[t_3-t_2]$$

$$\text{Inv}_4 = \int_{t_3}^{T} I(t)dt = \left(\frac{S_0\theta+D+\gamma.S_0}{\theta^2}\right)\left[-e^{-\theta.T}.e^{\theta.t_3}+e^{-\theta.t_3}.e^{\theta.t_3}\right]-\left(\frac{D+\gamma.S_0}{\theta}\right)[T-t_3]$$

$$\text{T - t}_3 = -\frac{1}{\theta}\log\left[\frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}\right] = \left[\frac{S_0}{\theta}\right]+\left(\frac{D+\gamma S_0}{\theta^2}\right)\log\left(\frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}\right) \quad \text{by}$$

(12)

The values of $\theta.S_0$ and $(D+\gamma S_0)$ must be st $\text{Inv}_4>0$ is satisfied. Now the total deteriorated items(Id) is

$$I_d = \theta\left\{\int_0^{t_1} I(t).dt + \int_{t_1}^{t_2} I(t).dt + \int_{t_2}^{t_3} I(t).dt + \int_{t_3}^{T} I(t).dt\right\}$$

$$= \theta\,(Inv_1 + Inv_2 + Inv_3 + Inv_4)$$

Therefore the total demand in (0,t) becomes $D_t = (a.t_2 - I_d)$ then the average profit during time (0,t) takes the form

$$\pi(P,t_2) = \frac{1}{T}\left[(at_2 - I_d)S_P - \left\{C_S + C_n(Inv_1+Inv_2+Inv_3+Inv_4) + \left(r+\frac{g}{a}+\alpha a\right)a.t_2\right\}\right] \text{--(13)}$$

Therefore we have to maximize $\pi(P,t_2)$

Subject to the constraints

$D+(\theta+\gamma)S_0 - a < 0$

- $-\text{Inv}_1 < 0$
- $-\text{Inv}_2 < 0$
- $-\text{Inv}_3 < 0$
- $-I(t_2) + S_0 < 0$
- $-t_2 + t_1 < 0$

The condition $D+(\theta+\gamma)S_0 - a < 0 \Rightarrow) < \dfrac{\theta.S_0}{a-D-\gamma.S_0} < 1$ which is necessary for the value of t_1 in eq^n (6) to be real.The three conditions $-\text{Inv}_1 < 0$, $-\text{Inv}_2 < 0$, $-\text{Inv}_3 < 0$ ensure that Inv_1, Inv_2, Inv_3 must be positive.The condition $-I(t_2) + S_0 < 0$ Implies that $I(t_2)$, the Inventory level at time t_2 is higher than S_0.The condition $-t_2+t_1<0$ ensures that t_2 is greater than t_1.This problem can be solved by Zoutrn disk method. Whose algorithm is discussed below.

General Problem : Minimize {$- -\pi(\overline{X})$} subject to the constraints : Gj($\overline{X}$) < 0, where $\overline{X} \in R^n$, j = 1, 2...........m.

ALGORITHM

1. Start with an initial feasible point $\overline{X}_1$, evaluate $\pi(\overline{X}_1)$ and Gj($\overline{X}_1$), j = 1, 2, ...m. Set the iteration number as i = 1.
2. If Gj($\overline{Xi}$) < 0, j = 1, 2, ...,m. (i.e., ($\overline{Xi}$) is an interior feasible point), set the current search direction as $\overline{S}_i = -\nabla\pi(\overline{Xi})$. Normalize $\overline{S}_i$ in a suitable manner.
3. Find a suitable step length λ_i along the direction $\overline{S}_i$ and obtain a new point $\overline{X}_{i+1}$ as $\overline{\boldsymbol{X}}_{\boldsymbol{i+1}} = \overline{X}_1 + \lambda_i\,\overline{S}_i$
4. Evaluate the objective function $\pi(\overline{X}_{i+1})$.

5. Test for the convergence of the method. If $| \{\pi(X_{i)} - \pi(X_{i+1})\} / \pi(X_i) | \leq \in$ where $\in$ is a pre assigned small positive quantity, terminate the iteration by taking $X_{opt} \approx X_{i+1}$. Otherwise, go to next step.

6. Set the new iteration number as i = i + 1 , and repeat from step 2 onwards.

NUMERICAL EXAMPLE

We take the parameter values as D = 50, _ = 0.05, = 0.1, S_0 = 100, C_s = 300, C_h = 0.1, S_p = 6.0, r = 1.0, g = 250, _ = 0.01 in appropriate units.

We obtain the optimum results t_1 = 1.258883, t_2 = 6.696204, t_3 = 10.07596, T = 11.67682, P= 141.9617

CONCLUSION

In this paper an inventory model is developed and analyzed for decaying items with stock -dependent demand rate. It is assumed that the demand rate remains stock-dependent for an initial period after which a uniform demand rate follows as the stock comes down to a certain level. The unit production cost is taken to be a function of the finite production rate which is treated to be a decision variable. The mathematical expression for the average profit function is derived and it is maximized subject to the different constraints of the system using method of constrained optimization, the algorithm of which is given. The solution procedure is illustrated with the help of a numerical example.

REFERENCES

1. A. C. Hax and D. Candea, Production and Inventory Management,Production and Inventory Management , Prentice-Hall, Englewood Cli_s,NJ ,(1984).
2. G. L. Alder and R. Nanda, The effects of learning on optimal lot size determination-single product case, AIIE Trans. 6 (1974) 14-20.
3. D. R. Sule, A note on production time variation in determining EMQ under influence of learning and forgetting,AIIE Trans. 13(1981) 91-95.
4. D. R. Sule, The e_ect of a alternate periods learning and forgetting on economic manufacturing quantity, AIIE Trans. 10 (1978) 338-343.
5. S. Axster and S. E. Elmaghraby, A note on EMQ under learning and forgetting,AIIE Trans. 13 (1981) 86-90.
6. E. J. Muth and K. Spearmann, Learning e_ects in economic lotsize,Mgmt Sci. 29 (1983) 264-269.
7. E. L. Proteus, Optimal lot sizing, process quality improvement, and setup cost reduction,Ops. Res. 34 (1986) 137-144.
8. M. J. Rosenblatt and H. L. Lee, Economic production cycles with imperfect production processes, IIE Trans. 17 (1986) 48-54.208
9. T. C. Cheng, An economic order quantity model with demand dependent unit production cost and imperfect production processes, IIE Trans.23 (1991) 23-28.
10. P. J. Schweitzer and A. Seidman, Optimizing processing rates for flexible manufacturing systems, Mgmt. Sci. 37 (1991) 454-466.
11. M. Khouja and A. Mehrez, An economic production lot size model with imperfect quality and variable production rate, Opl. Res. Soc.45 (1994) 1405-1417.
12. M. Khouja, The economic production lot size model under volume flexibility, Computers Ops. Res. 22 (1995) 515-523.
13. E. A. Silver, Deliberately slowing down output in a family production context, Int. J. Prod. Res. 28 (1990) 17-27.
14. I. Moon, G. Gallego and D. Simchi-levi, Controllable production rates in a family production context, Prod.Res.29 (1991) 2459-2470.
15. G. Gallego, Reduced production rates in the economic lot scheduling problem, Int. J. Prod. Res. 31 (1993) 1035-1046.
16. E. A. Silver and R. Peterson, Decision Systems for Inventory Management and Production Planning, Wiley, New York, 2nd edn.(1985) 17. E. A. Silver, A simple inventory replenishment decision rule for a linear
17. trend in demand, Opl. Res. Soc.30 (1979) 71-75.
18. R. C. Baker and T. L. Urban, A deterministic inventory system with an inventory-level-dependent demand rate, Opl.Res. Soc.39 (1988) 823-831.
19. T. L. Urban, An inventory model with an inventory-level-dependent demand rate and relaxed terminal condition, Opl. Res. Soc. 43 (1992) 721-724.
20. M. Goh, EOQ models with general demand and holding cost functions,Euro. J. Oper. Res.73 (1994) 50-54.

21. B. N. Mondal and S. Phaujder, An inventory model for deteriorating items and stock-dependent demand rate, Opl. Res. Soc. 40 (1989) 483-488.209
22. T. K. Datta and A. K. Pal, A note on an inventory model with inventory level-dependent demand rate, Opl.Res. Soc. 41 (1990) 971-975.
23. G. Zoutendijk, Nonlinear Programming: A Numerical Survey, SIAM J. Control. 4 (1966) 194-210.
24. B. C. Giri, S. Pal, A. Goswami and K. S. Chaudhuri, An inventory model for deteriorating items with stock-dependent demand rate, Euro.J.Oper. Res.95 (1995) 604-610.
25. J. Roy and K. S. Chaudhuri, An EOQ model with stock-dependent demand, shortages, inflation and time discounting, Int. J. Prod. Eco. 53 (1997) 171-180.
26. J. Roy, A. Goswami and K. S. Chaudhuri, On an inventory model with two levels of storage and stock-dependent demand rate, Int. J.Systems. Sci.29 (1998) 249-254.
27. B. C. Giri and K. S. Chaudhuri, Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost, Euro. J. Oper. Res.105 (1998) 467-474.
28. A. K. Sethi and P. S. Sethi, Flexible in Manufacturing: A Survey, Int. J. Flexible Manufact. Systems.2 (1990) 289-328.
29. Bansal K.K., Ahlawat N. [2012] Integrated Inventory Models for Decaying Items with Exponential Demand under Inflation. *International Journal of Soft Computing and Engineering (IJSCE) 2[3] 578-587*
30. Bansal K.K. [2016] A Supply Chain Model With Shortages Under Inflationary Environment. *Uncertain Supply Chain Management* 4[4] 331-340
31. Sharma M.K.,Bansal K.K.[2017] Inventory Model for Non-Instantaneous Decaying Items with Learning Effect under Partial Backlogging and Inflation. *Global Journal of Pure and Applied Mathematics.vol.13 [6]* pp. 1999-2008
32. Kumar A., Bansal K.K.[2014] A Deterministic Inventory Model for a Deteriorating Item Is Explored In an Inflationary Environment for an Infinite Planning Horizon. *International Journal of Education and Science Research Review1 [4] 79-86*
33. Yang, G., Ronald, R.J. and Chu, P. (2005), Inventory models with variable lead time and present value,European Journal of Operational Research,164, 358-366
34. Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006), An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging, International Journal of Production Economics, 101, 369-384.
35. Yang, H.L. (2006), Two-warehouse partial backlogging inventory models for deteriorating items under inflation, International Journal of Production Economics, 103(1), 362-370
36. Teng, J.T., Ouyang, L.Y. and Chen, L.H. (2007), A comparison between two pricing and lot-sizing models with partial backlogging and deteriorating items, International Journal of Production Economics, 105(1), 190-203.
37. Lo, S. T., Wee, H. M. and Huang, W. C. (2007), An integrated production inventory model with imperfect PRODUCTION process and Weibull distribution deterioration under inflation, International Journal of Production Economic, 106, 493-505.
38. Roy, A, (2008). An inventory model for deteriorating items with price dependent demand and time varying holding cost. *Advanced Modeling and Optimization*, 10, 25-37
39. Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009). Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research*, 192, 79-92.
40. He, Y., Wang, S.-Y., & Lai, K.K. (2010). An optimal production-inventory model for deteriorating items with multiple-market demand. *European Journal of Operational Research*, 203(3), 593-600.
41. Dye, C.-Y., & Ouyang, L.-Y. (2011). A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing. *Computers & Industrial Engineering*, 60(1), 127-137.
42. Leea Y.-P., & Dye, C.-Y. (2012). An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. *Computers & Industrial Engineering*, 63(2), 474-482.
43. Maihami, R., & Kamalabadi, I.N. (2012). Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. *International Journal of Production Economics*, 136(1), 116-122.